Phys 410
Spring 2013
Lecture #10 Summary
13 February, 2013

We considered energy for motion in curvilinear one-dimensional systems. An example is a car moving on a roller coaster track. Consider a particle confined to move along a one-dimensional 'track' parameterized by its displacement s from some arbitrary origin. It has a kinetic energy $T = \frac{1}{2} m\dot{s}^2$. The kinetic energy can be altered by applying a tangential force and doing work on the particle. Newton's second law can be stated as $m\ddot{s} = F_{net}$. If the tangential force is conservative, then you can define a potential energy U, and a total mechanical energy $E = T + U$.

We next considered central forces. These are forces that are everywhere directed toward a fixed force center. Such a force has the form $\vec{F}(\vec{r}) = f(\vec{r})\hat{r}$. If further the force is spherically symmetric, then the scalar function depends only on the radial distance and not the angular coordinates: $f(\vec{r}) = f(r)$.

There are two statements that can be made about central forces:

1) A central force that is conservative is automatically spherically symmetric,
2) A central force that is spherically symmetric is automatically conservative.

We proved the first of these two statements. If the force is conservative, then it can be represented in terms of the gradient of a scalar potential: $\vec{F} = -\vec{\nabla} U(\vec{r})$. Using the gradient in spherical coordinates, derived in class ($\vec{\nabla} = \hat{r}\partial / \partial r + (\hat{\theta} / r)\partial / \partial \theta + (\hat{\phi} / r \sin\theta)\partial / \partial \phi$), we find that a central force (dependent on $\hat{r}$ only) requires that $\partial U / \partial \theta = \partial U / \partial \phi = 0$. This means that the potential energy depends only on the radial coordinate: $U = U(r)$. In turn, the central force can only depend on the scalar radial coordinate: $\vec{F} = -\hat{r}\ \partial U(r) / \partial r$, which means that it is spherically symmetric (i.e. no dependence of the potential and force on the angular coordinates θ, ϕ). The one-dimensional nature of the potential energy and force will have benefits later when we look at the two-body problem.